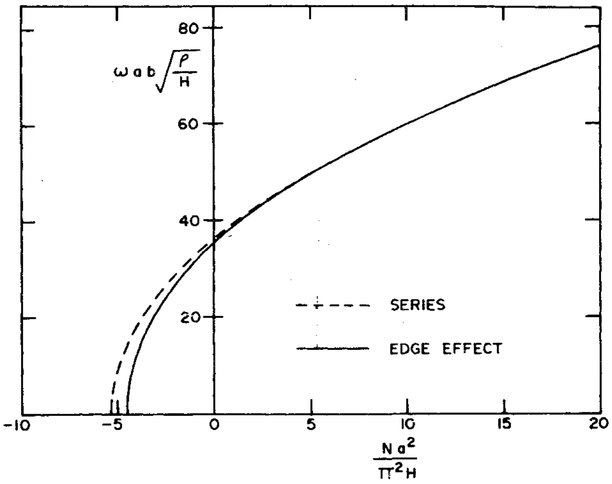

Fig. 1 Graph of fundamental frequency parameter vs in-plane load for square isotropic plate.

Table 2 Hydrostatic buckling parameter $Na^2/\pi^2 D$ for a square clamped orthotropic plate (compression positive)

	D_x/H		
D_y/H	1/2	1	2
1/2	2.91		
1	3.72	4.50 (5.31)[a]	
2	5.20	6.07	7.76

[a] ( ) accurate result by Taylor.[5]

it may be recognized that the closer a plate deflection comes to a sine wave, the smaller the edge effect error and thus the greater the accuracy achieved. This is borne out by the results of Table 1 where it may be seen that the larger discrepancies occur for the compressive forces, where the plate deflection tends to deviate from a sine wave, than for tensile forces, where the plate tends to behave more and more like a membrane, the exact shape for which is a pure sine wave. Figure 1, which shows the variation in frequency with in-plane load for the isotropic plate (computed using the edge effect method and the series solution), further illustrates this trend.

For completeness, Table 2 shows the buckling loads obtained using the edge effect method for the various cases of orthotropy and, in the case of the isotropic plate, shows the accurate value presented by Taylor.[5] The accuracy of the method for buckling problems is somewhat poorer than that for natural frequency calculations for plates subject to zero or tensile in-plane forces. But, since the method is believed to always yield a lower bound for single plates (which is not necessarily the case for plate systems[3]), the results still have utility.

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Separation-Like Similarity Solutions on Two-Dimensional Moving Walls

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Introduction

The objective of the following Note is to present the results of a study of two-dimensional separation-like velocity profiles on moving walls based on similarity solutions of the incompressible boundary-layer equations. The underlying motivation is the possibility of using similarity solutions as a family of velocity profiles in an integral theory for the nonsimilar boundary-layer development on moving walls. In such a method it is important to identify the profile characteristics associated with separation.

Prandtl[1] defined the important features of an incipient separation velocity profile on a fixed wall in his historic 1904 paper. Separation is preceded by vanishing wall shear stress so that the criterion for separation is $\partial u/\partial y = 0$, $y = 0$.

The Falkner-Skan group of similarity solutions contains a profile which meets this criterion and when the similarity profiles and potential flow pressure distribution are employed in an integral theory separation is predicted at 105.2° on a circular cylinder. This compares with 104.5° predicted by numerical techniques.[2]

The boundary-layer on a moving wall is dominated by the conditions near the surface which prevents the fixed wall form of separation. Application of Prandtl's criterion results in a layer of fluid moving with the wall and not separation. Moore[3] modified Prandtl's criterion as follows:‡

$$\partial u/\partial y = 0, \qquad u = 0 \tag{1}$$

at a coincident point which includes Prandtl's criterion as a special case. When the wall is moving in the same direction as the boundary-layer edge velocity, Moore's criterion requires that the fluid be brought to rest by the adverse pressure gradient at an intermediate point in the boundary layer. Downstream of this point is an imbedded region of reverse flow.

Solutions to the similarity equations which meet this criterion were obtained by Moore for two wall velocities. Additional examples have been computed so that the variation of β, the pressure gradient parameter, and other integral shape factors for this class of separation profiles can be defined.

Equations

The incompressible, two-dimensional boundary-layer equations are transformed into the similarity equation for the

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‡ N. Rott and W. Sears also reached the same conclusion in unpublished work.

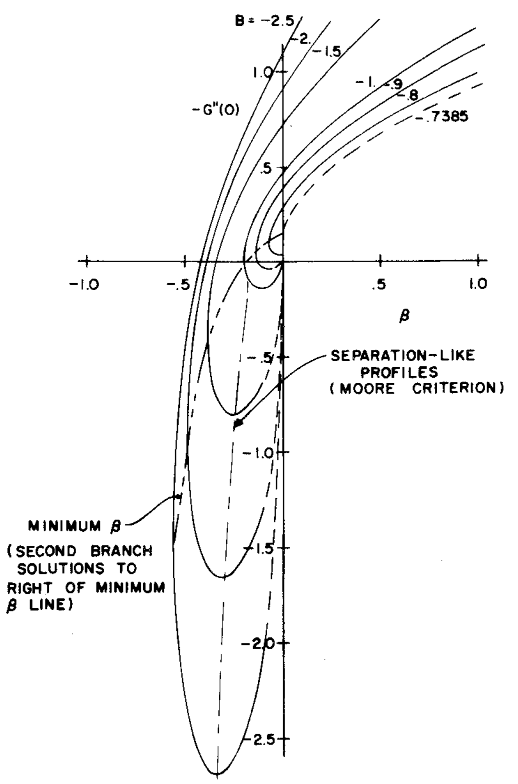

Fig. 1 $G''(0)$ **vs** β **with** B **as parameter** ($AB > 0$, $B < 0$).

moving wall case by assuming $G'(\eta) = (u-u_e)/(u_w-u_e)$, $\eta = y/h(x)$, so that the equations of motion become

$$G''' + A[(G+B\eta)G'' - \beta(G'^2 + 2BG')] = 0 \tag{2}$$

with boundary conditions

$$G(0) = G'(\infty) = 0, \qquad G'(0) = 1$$

Similarity of the velocity profile results if

$$A = \pm 1$$
$$B = u_e/(u_w - u_e)$$
$$u_e \sim u_w \sim x^{\beta/(2-\beta)}$$
$$h = [(2-\beta)AB\nu x/u_e]^{1/2}$$

Further details on the derivation and solutions of Eq. (2) are found in Ref. 4.

Equation (2) was numerically integrated using a variable step Runga-Kutta technique and a modification of the method of Nachtsheim and Swigert.[5] Their method can be used to determine values of $G''(0)$ and β that minimize the mean-square derivative of G' and G'' at some large value of η from the asymptotic boundary conditions. The modification introduced included the minimization of the mean-square deviation of G' and G'' from zero at some intermediate point within the boundary layer. This is equivalent to imposing Moore's criterion for separation.

Results

The separation-like profiles that have been obtained belong to the second branch of the similarity solutions. That is, at a given value of the wall to edge velocity ratio (Constant B) the solutions are double valued in $G''(0)$ for $\beta < 0$. Figure 1 shows a number of $-G''(0)$ vs β curves for various values of B. Negative of $G''(0)$ is used to conform with the usual presentation based on the Falkner-Skan solutions. The Falkner-Skan separation profile corresponds to $G''(0) = 0$ at $\beta = -0.19884$ on the $B = -1$ curve. It coincides with the minimum β point for the non-moving wall case. As the wall velocity is increased ($B < -1$), the separation profile becomes more closely associated with the minimum value of $-G''(0)$ and beyond $B = -1.5$ the solutions fall on the minimum.

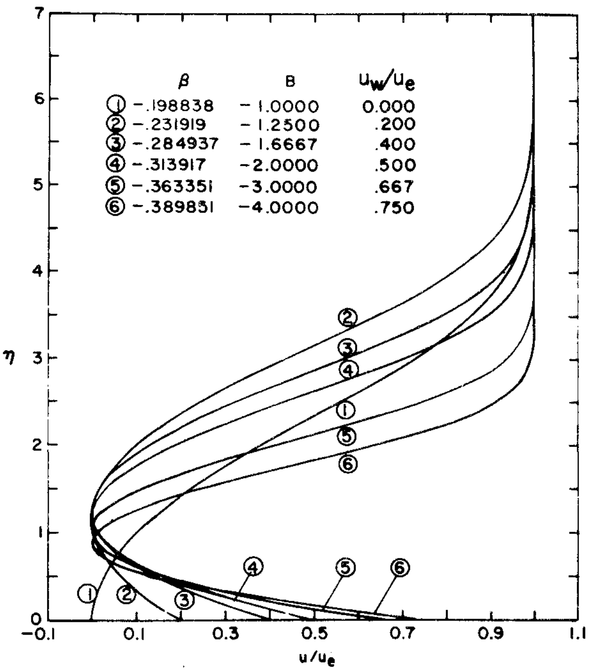

Fig. 2 Velocity profiles for separated flow ($B \leq -1$).

Figure 2 shows several velocity profiles for the separation-like solutions. Figure 3 contains a plot of β for separation-like profiles vs u_w/u_e. The decrease in β with increasing wall velocity is an indication of the increasing adverse pressure gradient required to separate the boundary-layer on a moving wall. No separation-like solutions were found when $u_w > u_e (B > 0)$. At large negative values of β and $B > 0$ the velocity profiles show an overshoot near the wall because of the strong wall deceleration rather than a pressure gradient effect.

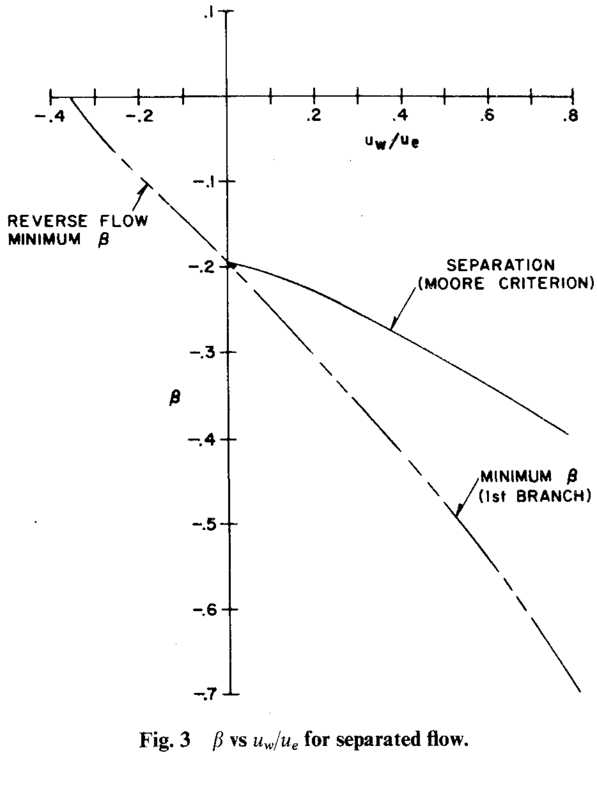

Fig. 3 β **vs** u_w/u_e **for separated flow.**

Moore has proposed that the same moving wall criterion [Eq. (1)] applies to reverse flow separation, i.e., when the wall velocity moves counter to the edge velocity. The boundary-layer equation does not permit such a solution because a point where $\partial u/\partial y = u = 0$ and the condition that $u_w < 0$ and $u_e > 0$ also implies $\partial^2 u/\partial y^2 = 0$. As a result the inertia and viscous terms are zero at that point and the pressure gradient term would be unbalanced. Thus, there are no velocity profiles of the kind suggested by Eq. (1). Examination of the reverse flow similarity profiles[4] shows that there are no special characteristics which can be associated with separation such as an inflection point or an infinite displacement thickness and there is no tendency toward such conditions.

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Strong Interaction Associated with Transonic Flow Past Boattailed Afterbodies

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In order to estimate the performance of air breathing propulsive systems of a high speed aircraft operating under subsonic cruise conditions, it is necessary that the flowfield associated with transonic flow past boattails can be successfully studied and analyzed. This type of problem usually appears to be extremely difficult as the governing inviscid flow equation is of the mixed type. In addition, the relatively short boattail is usually immersed within the thick boundary layer of the approaching upstream flow so that the viscous interaction coupling the viscous and its external flows cannot be disregarded. Experimental investigation of such flows has been carried out, e.g., by Shrewbury.[1] His experimental data indicated considerable influence of the boattail juncture shape to the pressure distribution on the afterbody especially at high subsonic free-stream Mach numbers which in turn illustrated the extremely sensitive character of the transonic flow.

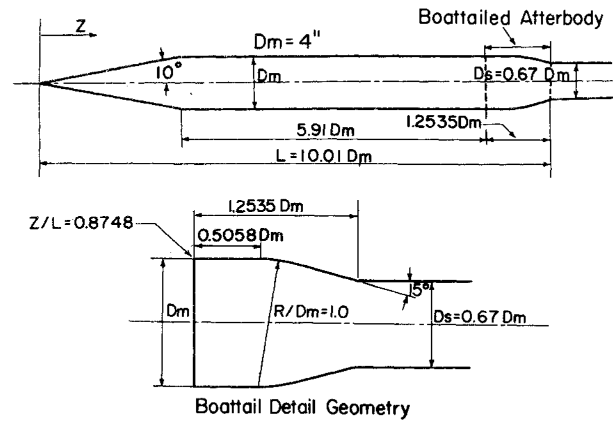

Fig. 1 Model geometry of the boattailed afterbody.

Since the suggestion of numerical treatment of inviscid transonic flow was made by Murman and Cole,[2] a considerable amount of activities in this area has been carried out. Krupp and Murman[3] computed transonic flow past lifting airfoils and slender bodies. Bailey[4] also extended the small disturbance treatment to calculate transonic flow past slender bodies of revolution. Steger and Lomax[5] and South and Jameson[6] employed the full potential equation to calculate the transonic flow past two-dimensional and axisymmetric bodies, respectively. It seems that the problem of transonic flow past boattails can be studied with these numerical relaxative schemes. Indeed, it is the intention here to report these results obtained from such a study. The preliminary calculations are restricted to a particular model configuration as shown in Fig. 1 which has been tested in the experimental program.[1] It was learned that the small disturbance treatment of the inviscid part of the transonic flow is not adequate even though the model appears to be relatively slender; thus, the full potential equation must be employed for its study. It will be seen that the "strong interaction" character of these problems within the transonic flow regime will be fully illustrated from the results obtained from this study even though the flow has not been separated away from the boattailed afterbody.

The detailed derivation and transformation of the basic equations for the inviscid, viscous flows and the description of numerical calculations will be presented in Ref. 7. Only a brief description of the method of calculations is presented here. For the invsicid flow, the basic equation for the disturbed velocity potential in the r, z coordinate system was transformed to[6]: $\xi = z$, $\zeta = B\eta/(1+B\eta)$ with $\eta = r - r_b(z)$ where B is a

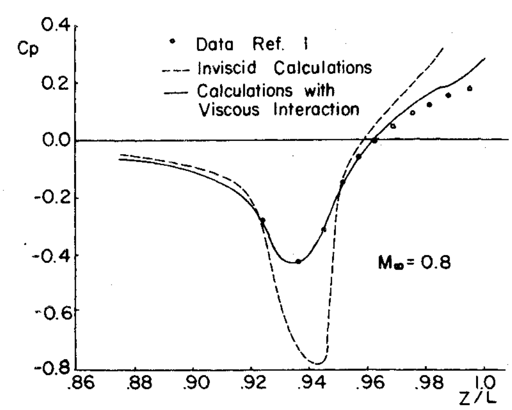

Fig. 2 Results of calculations and their comparison with the experimental data ($M_\infty = 0.8$).

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